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STABILITY OF A BODY STABILIZED BY FINS AND SUSPENDED FROM AN AIRPLANE

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SUMMARY

A theoretical investigation has been made of the oscillations performed by suspended bodies of the type commonly used for trailing airspeed heads and similar towed devices. The primary purpose of the investigation was to design an instrument that will remain stable as it is drawn up to a support underneath an airplane without attention on the part of the pilot. Flight tests of a model airspeed head were made to supplement the theoretical study. Unstable oscillations of the body at short cable lengths were predicted by the theory, but the rate of increase of amplitude of these oscillations was very small. In flight tests, more violent types of instability were believed to be caused by unsteady or nonuniform air flow in the region where the cable was lowered from the airplane. No practical method was found to provide large damping of the oscillations at short cable lengths, but the degree of stability present in a suitably designed suspended body was shown to be satisfactory if the body was lowered into a uniform air stream.

INTRODUCTION

Suspended devices that consist of heavy streamline bodies stabilized by fins have been used in the past for various purposes. A frequent application of this type of device is the suspended airspeed head used for the accurate measurement of airplane speed (reference 1). Certain difficulties have been encountered in the use of these instruments because of unstable oscillations of the cable and suspended body. One common type of instability has been a tendency of the instrument to swing violently back and forth and from side to side as it was drawn up close to the airplane. Because of this

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tendency, these instruments need considerable attention in handling and usually require the services of a person other than the pilot. Another type of instability has been an oscillation of the body and whipping action of the cable when the body was being towed at the full length of the cable. This motion has occurred only when the instrument was lowered from certain airplanes.

The present investigation was undertaken in an effort to develop a type of trailing airspeed head that could be lowered from and drawn up to a support underneath the airplane without close attention. This requirement necessitates that unstable oscillations of the instrument be avoided at any cable length.

A study of both lateral and longitudinal oscillations of the body and cable system was made in reference 2. This study was based on the assumption that the damping of the motion due to air forces on the cable could be neglected. The present investigation shows that this assumption leads to erroneous conclusions with regard to the boundaries of stability.

SYMBOLS

m mass of towed body

y lateral displacement

C_y coefficient of side force on vertical tail

y_c lateral displacement of cable element

ψ angle of yaw

N yawing moment

β angle of sideslip

V forward velocity

l tail length

Y lateral force

C moment of inertia about vertical axis

a slope of tail lift curve (negative) $\left(\frac{dC_y}{d\beta}\right)$

S vertical-tail area

ρ air density

Z vertical distance between airplane and body

z vertical distance

g acceleration of gravity

D effective drag; also, operator indicating differentiation with respect to time $\left(\frac{d}{dt}\right)$

D' nondimensional operator (DT)

D_b drag of body

D_c total cable drag

C_{D_b} drag coefficient of body $\left(\frac{D_b}{\frac{\rho}{2}V^2 S}\right)$

C_{D_c} equivalent drag coefficient of cable $\left[\frac{Kw_c Z}{S} \int_0^1 c_{d_c} \frac{z}{Z}\, d\left(\frac{z}{Z}\right)\right]$

C_D effective drag coefficient $\left(C_{D_b} + C_{D_c}\right)$

X horizontal distance between airplane and body

c_{d_c} drag coefficient of cable per unit vertical height $\left(\frac{\text{cable drag}}{\frac{\rho}{2}V^2 w_c z}\right)$

F Froude number $\left(\frac{V^2}{gl}\right)$

μ relative-density coefficient $\left(\frac{m}{\frac{\rho}{2}Sl}\right)$

R ratio of tail length to vertical distance below airplane (l/Z)

C_1 moment-of-inertia factor $(k/l)^2$

k	radius of gyration
t	time
T	time unit $\left(\frac{m}{\frac{\rho}{2}VS}\right)$
a,b,c,d,e	coefficients of quartic
w_c	cable diameter
ϕ	angle of cable with horizontal
K	fraction of cable side force applied to body
Y_b	side force on body
Y_c	side force on cable
P	period of oscillation
$N_{1/2}$	number of cycles to damp to one-half amplitude

$$Y_\beta = \frac{1}{m}\frac{\partial Y}{\partial \beta}$$

$$Y_y = \frac{1}{m}\frac{\partial Y}{\partial y}$$

$$Y_{\dot{y}} = \frac{1}{m}\frac{\partial Y}{\partial \dot{y}}$$

$$Y_{\dot{\psi}} = \frac{1}{m}\frac{\partial Y}{\partial \dot{\psi}}$$

$$N_\beta = \frac{1}{C}\frac{\partial N}{\partial \beta}$$

$$N_{\dot{\psi}} = \frac{1}{C}\frac{\partial N}{\partial \dot{\psi}}$$

A dot over a symbol indicates the first derivative of the quantity with respect to time t and two dots over a symbol indicates the second derivative.

THEORETICAL INVESTIGATION

Because of the axial symmetry of a trailing airspeed head, its longitudinal and lateral motions occur independently. These types of motion may, therefore, be treated separately. The lateral motion of the instrument will be analyzed in considerable detail because this mode of motion is theoretically most likely to become unstable.

Lateral Oscillations

Mathematical treatment is possible only for the case of small oscillations, for which the forces acting on the body vary linearly with the displacements and angular velocities. The instrument will swing from side to side like a pendulum but, for small amplitudes, its motion may be considered to take place in a horizontal plane. A restoring force depending on the cable length under consideration will be assumed to act through the pivot point.

The subsequent analysis indicates that the drag force on the cable and body has an important influence on the damping of the oscillations. In practice, almost all the drag acts on the cable. For purposes of analysis, however, an effective drag force due to the cable will be assumed to act on the body at its center of gravity. The relation between this effective drag force and the characteristics of the cable will be discussed later.

The notation used in considering the lateral motion is shown in figure 1. The equations of motion with respect to a fixed system of axes are as follows:

$$m\ddot{y} - \beta \frac{\partial Y}{\partial \beta} - y \frac{\partial Y}{\partial y} - \dot{y} \frac{\partial Y}{\partial \dot{y}} - \dot{\psi} \frac{\partial Y}{\partial \dot{\psi}} = 0$$

$$C\ddot{\psi} - \beta \frac{\partial N}{\partial \beta} - \dot{\psi} \frac{\partial N}{\partial \dot{\psi}} = 0$$

$$\frac{\dot{y}}{V} - \psi = \beta$$

In order to simplify the notation, let $D = \frac{d}{dt}$ and define the stability derivatives

$$Y_\beta = \frac{1}{m} \frac{\partial Y}{\partial \beta}$$

$$N_\beta = \frac{1}{C} \frac{\partial N}{\partial \beta}$$

and define the other derivatives similarly. These equations may be solved by the usual procedure of setting the determinant of the coefficients equal to zero. This determinant may be expanded to give the quartic

$$aD^4 + bD^3 + cD^2 + dD + e = 0 \qquad (1)$$

where

$$a = 1$$

$$b = -N_{\dot{\psi}} - \frac{Y_\beta}{V} - Y_{\dot{y}}$$

$$c = N_\beta - Y_y + \frac{Y_\beta}{V} N_{\dot{\psi}} + Y_{\dot{y}} N_{\dot{\psi}} - \frac{N_\beta}{V} Y_{\dot{\psi}}$$

$$d = -N_\beta Y_{\dot{y}} + Y_y N_{\dot{\psi}}$$

$$e = -Y_y N_\beta$$

In order to find the nature of the motion from equation (1), it is necessary to evaluate the stability derivatives in terms of the dimensions and aerodynamic characteristics of the instrument. In setting up a simplified form for the stability equation, it is sufficiently accurate to assume that aerodynamic forces other than drag forces will act only on the vertical fin of the instrument. The derivation of the expression for Y_β is given as an example:

$$Y = \beta \frac{\partial Y}{\partial \beta}$$

$$= \beta a \frac{\rho}{2} V^2 S$$

where

$$a = \frac{dC_Y}{d\beta}$$

$$\frac{\partial Y}{\partial \beta} = a \frac{\rho}{2} V^2 S$$

$$Y_\beta = \frac{1}{m} \frac{\partial Y}{\partial \beta}$$

$$= \frac{a \frac{\rho}{2} V^2 S}{m}$$

The other aerodynamic derivatives may be determined in a similar manner. Only the derivatives related to the forces exerted by the cable require special consideration. The Y-force caused by a lateral displacement of the body is found by assuming that the body and cable system, when viewed from the front, deflects as a simple pendulum (fig. 2). The restoring force due to a small deflection y of the body suspended a vertical distance Z below the airplane is

$$Y = -\frac{mgy}{Z}$$

$$\frac{\partial Y}{\partial y} = -\frac{mg}{Z}$$

$$\begin{aligned} Y_y &= \frac{1}{m}\frac{\partial Y}{\partial y} \\ &= -\frac{g}{Z} \end{aligned}$$

The derivative $Y_{\dot{y}}$ is found from the drag force acting on the body and cable. A drag force acting on the body will have a component of side force as shown in figure 3. Thus

$$\begin{aligned} Y &= D_b(\beta + \psi) \\ &= \frac{D_b\dot{y}}{V} \\ &= C_{D_b}\frac{\rho}{2}V^2S\frac{\dot{y}}{V} \end{aligned}$$

$$\frac{\partial Y}{\partial \dot{y}} = C_{D_b}\frac{\rho}{2}VS \tag{2}$$

The component of side force due to the body is

$$\begin{aligned} Y_{\dot{y}} &= \frac{1}{m}\frac{\partial Y}{\partial \dot{y}} \\ &= C_{D_b}\frac{\frac{\rho}{2}VS}{m} \end{aligned}$$

Inasmuch as the drag of the cable ordinarily far exceeds the drag of the body, the value of the derivative $Y_{\dot{y}}$ will be principally determined by the cable drag. The method of calculating an equivalent drag coefficient C_{D_c} to take into account the effect of the cable is given in the appendix. The derivative $Y_{\dot{y}}$ is then given as follows:

$$Y_{\dot{y}} = \left(C_{D_b} + C_{D_c}\right)\frac{\frac{\rho}{2}VS}{m}$$

$$\equiv C_D \frac{\frac{\rho}{2}VS}{m}$$

The value of the coefficient C_{D_c} may be determined as a function of the ratio of horizontal length to height of the cable X/Z from figure 4.

All the aerodynamic derivatives have been evaluated in terms of the dimensions of the body and cable system. In order to reduce the number of variables, it is convenient to express these derivatives in terms of nondimensional ratios of the quantities involved, which are given as follows:

Froude number,

$$F = \frac{V^2}{gl}$$

Relative-density coefficient,

$$\mu = \frac{m}{\frac{\rho}{2}Sl}$$

Ratio of tail length to vertical distance of body below point of support,

$$R = \frac{l}{Z}$$

Moment-of-inertia factor,

$$C_1 = \left(\frac{k}{l}\right)^2$$

Time is expressed in terms of the time unit $T = \frac{m}{\frac{\rho}{2}VS}$.

When the derivatives are expressed in terms of these variables, the stability quartic becomes

$$aD'^4 + bD'^3 + cD'^2 + dD' + e = 0 \qquad (3)$$

where

$$D' = DT$$

and

$$a = 1$$

$$b = \frac{a}{C_1} - a + C_D$$

$$c = \frac{a\mu}{C_1} + \frac{\mu^2 R}{F} - \frac{aC_D}{C_1}$$

$$d = \frac{a\mu C_D}{C_1} - \frac{a\mu^2 R}{C_1 F}$$

$$e = -\frac{a\mu^3 R}{C_1 F}$$

The stability of the towed airspeed head may be determined by substituting numerical values in the formulas for the coefficients and factoring the quartic. The two quadratic factors determine the period and damping of two modes of oscillation. One quadratic factor yields values of the period and damping very close to those obtained with a simple pendulum having a length equal to the vertical distance of the towed body below the airplane and damping equal to that supplied by the drag force. The other quadratic factor gives an oscillation that has values of period and damping very close to those of the body rotating as a weather vane about a vertical axis through its center of gravity. The weather-vane oscillation generally has a short period and is always rapidly damped. The damping of the pendulum oscillation is, however, very slow at short cable lengths, because the cable drag is small.

The coupling between the two modes of oscillation introduces the possibility of instability of the pendulum oscillation. In order to find the conditions for instability, the coefficients of the quartic may be substituted in Routh's discriminant, which states that the motion will be stable if the coefficients satisfy the relation

$$(bc - ad)d - b^2 e > 0 \tag{4}$$

When the values of the coefficients (equation (3)) are substituted in formula (4), Routh's discriminant becomes

$$\frac{1}{F^2}\left(\mu^3R^2 - \frac{C_D\mu^3R^2}{a}\right) + \frac{1}{F}\left(-\frac{a\mu RC_D}{C_1^{\,2}} - \frac{aC_D\mu R}{C_1} + \frac{\mu RC_D^{\,2}}{C_1} + \frac{a\mu^2R}{C_1}\right.$$

$$\left. - \frac{2\mu^2C_DR}{C_1} - \mu^2C_DR + a\mu^2R\right) - \frac{a\mu C_D}{C_1^{\,2}} - \frac{aC_D^{\,2}}{C_1^{\,2}} - \frac{a\mu C_D}{C_1}$$

$$-\frac{aC_D^{\,2}}{C_1} + \frac{C_D^{\,3}}{C_1} > 0$$

The expression is given in this form merely for the sake of completeness. In practice, a great simplification may be made, with negligible loss of accuracy, by neglecting the small term $-\frac{aC_D}{C_1}$ in coefficient c, formula (3). The simplified form of the discriminant is

$$\frac{F}{R\mu} > \frac{C_D + \frac{2C_D}{C_1} - \frac{a}{C_1} - a \pm \left(C_D - a - \frac{a}{C_1}\right)}{-\frac{2aC_D}{C_1}\left(1 + \frac{1}{C_1}\right)}$$

The minus sign before the expression $C_D - a - \frac{a}{C_1}$ gives one condition for stability

$$\frac{F}{R\mu} < -\frac{1}{-a\left(1 + \frac{1}{C_1}\right)} \qquad (5)$$

and the plus sign before the same expression gives another condition for stability

$$\frac{F}{R\mu} > \frac{C_1(C_D - a)}{-aC_D} \qquad (6)$$

Boundaries of stability are plotted in figure 5. It is seen that below a certain small value of the parameter $F/R\mu$, given by formula (5), the motion is stable for all values of the drag coefficient. As $F/R\mu$ is increased above this value, the motion is unstable until the boundary of stability given by formula (6) is reached. The motion then becomes stable again at all higher values of $F/R\mu$.

Examples have been worked out from the general boundaries of stability (fig. 5) to show the variation of stability of an actual airspeed head as the cable length is changed. The characteristics of the airspeed head and cable used in the calculations are as follows:

m, slug	0.466
l, foot	0.45
S, square foot	0.25
k, foot	0.416
Aspect ratio	2.25
a, per radian	-2.10
C_1	0.855
w_c, inch	0.375
Cable weight, pound per foot	0.05
ρ, slug per cubic foot	0.00238

The cable length is plotted against the effective drag coefficient $C_D = C_{D_b} + C_{D_c}$ in figure 6. The method for determining this curve is given in the appendix. The boundaries of stability for this particular case are plotted in the same figure in order that the region of instability may be found. As the body is lowered from the airplane, it will be stable for a very short distance and will then become unstable until the upper boundary of stability is reached. The upper boundary of stability occurs when the body is 2.8 feet below the point of support at an airspeed of 200 feet per second, or 6.3 feet below at 100 feet per second. For all greater cable lengths, the body will be stable. When the body is drawn up to the airplane, it will again pass through the unstable region.

The period and degree of damping of the oscillation at various cable lengths for the airspeed head having the characteristics previously given have been calculated by substituting numerical values in formula (3) and are given in the following table for an airspeed of 100 feet per second:

Distance below airplane, Z (ft)	Pendulum oscillation		Weather-vane oscillation	
	P (sec)	N	P (sec)	$N_{1/2}$
4.0	2.22	23.4 to double amplitude	1.08	3.87
6.3	2.86	∞ (neutrally stable)	1.07	4.10
10.0	3.51	20.0 to one-half amplitude	1.06	4.28
20.0	4.95	4.95 to one-half amplitude	1.07	4.34

From these calculations it is seen that the damping or rate of divergence of the pendulum oscillation is very small for cable lengths some distance on either side of the stability boundary. For the longest cable length, however, the oscillation damps to one-half amplitude fairly rapidly.

The boundaries of stability determined theoretically are in good qualitative agreement with the observed behavior of the NACA trailing airspeed head. Actually, there is no sharply defined boundary of stability because the oscillation is only slightly damped after the body has been lowered some distance into the stable region. As will be explained later, disturbing influences not taken into account in the theory may cause an unstable oscillation of the body when it would theoretically have a slightly damped oscillation.

The boundaries of stability shown in figure 5 indicate that, when the drag coefficient is zero, the body will be unstable at all values of cable length greater than that corresponding to the lower stability boundary. For very small values of the drag coefficient, such as would be obtained by neglecting the cable drag, the theory indicates that the body will be unstable over a large range of values of the cable length. The results of reference 2, in which the damping effect of air forces on the cable is neglected, are therefore believed to be in error.

Investigation of Modifications to Improve Stability

In order to investigate the changes that might be made to improve the stability of a conventional type of airspeed head, it is convenient to express the condition for stability (formula (6)) in the following form, where the nondimensional expressions have been removed by substituting the dimensional quantities that they replace:

$$\frac{1}{mg\frac{l}{Z}} > \frac{k^2/l^2}{-a\frac{\rho}{2}V^2S} + \frac{k^2/l^2}{D} \tag{7}$$

where k is the radius of gyration about an axis through the pivot point and D is the effective drag obtained by multiplying $C_D = C_{D_c} + C_{D_b}$ by $\frac{\rho}{2}V^2S$.

The curves of figure 5 show that the region of instability for a conventional type of towed body can never be entirely eliminated. The following changes would tend to restrict the unstable region to a region closer to the airplane:

(a) Decrease in weight mg

(b) Increase in drag

(c) Increase in area and aspect ratio of fin

(d) Decrease in ratio of radius of gyration to tail length k/l

The first two changes are impractical because they interfere with the usefulness of the instrument as an airspeed measuring device. The second two changes, however, provide practical methods of improvement. For example, the greatest distance below the airplane at which unstable oscillations occur in the example previously given could be decreased from 6.3 feet to 4.2 feet by doubling the tail length without increasing the radius of gyration. This change could be accomplished by mounting a light set of fins on a boom behind the instrument. Formula (7) indicates that increasing the speed will restrict the unstable region to shorter cable lengths. Once the oscillation becomes unstable, however, it will probably increase in amplitude faster at higher airspeeds. It may be advantageous, therefore, to raise and lower the body at low flying speeds.

The use of special devices to improve the stability will now be considered. It has been found by the writer that the two modes of oscillation given by a quartic will damp to one-half amplitude in the same time if the coefficients satisfy the relation

$$\frac{b^3}{8} - \frac{bc}{2} + d = 0$$

Such a condition will give the optimum use of damping in the system. Through examination of the coefficients of the quartic, formula (1), it is found that this relation may be satisfied by greatly increasing the damping in yaw $N_{\dot{\psi}}$ or by reducing the directional stability N_β almost to zero. Physically, a condition is thus reached at which the body remains approximately parallel to the average direction of flight as it swings from side to side instead of turning into the relative wind. Forces are thereby brought into play to damp out the pendulum oscillation.

The foregoing method of obtaining stability may also be explained in terms of the stability boundaries plotted in figure 5. In the small stable region below the lower boundary of stability, the pendulum oscillation is damped out by the mechanism just described. By reducing the directional stability and increasing the damping in yaw, the lower boundary of stability is raised to higher values of $F/R\mu$. It is theoretically possible, by using special devices that arbitrarily increase the damping in yaw or reduce the directional stability, to raise this stability boundary so that the unstable region is eliminated. It will be noted that this method of improving stability is different in principle from the one described following formula (7). The method based on formula (7) consisted in extending the stable region by lowering the upper boundary of stability. The method now being considered consists in raising the lower stability boundary.

It has been found impossible, in practice, to reduce the directional stability of a conventional towed body to the extremely small value required. The body of the instrument is generally unstable and some fin area is required to give neutral directional stability. Any small change in the characteristics of the body due to Reynolds number or due to small changes in shape would be sufficient to make it either directionally unstable or too stable to obtain damping by virtue of its low directional stability. The alternative, greatly increasing the damping in yaw, might be accomplished by operating the rudder of the instrument by means of a gyroscopic element to cause the rudder to deflect an amount proportional to the yawing velocity. The complication introduced by such a mechanism would probably make the method impracticable.

Another method of increasing the damping in yaw and at the same time reducing the directional stability is to use two fins, one at the front and one at the rear of the body. Calculations show that the directional stability must be reduced to a very small value (approximately 4 percent of the stability contributed by the rear fin) in order to avoid the unstable oscillations. A moderate decrease in directional stability, even when combined with a damping in yaw of 20 times that for a conventional body, will not avoid the unstable oscillations. If the required small directional stability could be obtained, any slight misalinement of the front and rear fins would cause the body to trim at a high lift coefficient. This condition would cause the body to fly out to one side and would also make it undesirable as an airspeed measuring device.

Longitudinal Oscillations

The longitudinal motion of a towed body has been treated theoretically in reference 2. This analysis neglected the damping of the motion contributed by air forces on the cable. The boundaries of stability calculated in reference 2 are therefore believed to be unconservative.

In practice, the fore-and-aft pendulum motion of the body has never been observed to become unstable at long cable lengths. It is noted that the effect of the cable could be taken into account as an equivalent drag coefficient, as it was for the lateral oscillations. If a value of drag coefficient of the correct order of magnitude is substituted in the relations presented in reference 2, the pendulum oscillations may be shown to be well damped at long cable lengths.

At short cable lengths and moderate speeds, the body hangs approximately vertically below the point of support; therefore, very little coupling exists between fore-and-aft movement of the body and pitching motion. The oscillation is simply a pendulum motion with damping supplied by the cable drag. Inasmuch as this cable drag is small at short cable lengths, the oscillation, though theoretically stable, is slowly damped and may become unstable if disturbing influences are present.

The analysis of reference 2 shows that other modes of longitudinal oscillation involving bowing of the cable and pitching of the body are theoretically possible, but such oscillations have never been observed in practice. It is believed that the drag on the cable prevents these oscillations from becoming unstable.

EXPERIMENTAL INVESTIGATION

Flight tests were made of an approximately $\frac{1}{2}$-scale model about dynamically similar to the NACA trailing airspeed head suspended from a Stinson SR-8E airplane. A drawing of the model is shown in figure 7. In order to simulate pulling the head up to a support under the airplane, the cord was run through an eyelet on the cabin steps.

The instrument was stable when towed on the end of a 75-foot cable at speeds between 80 and 150 miles per hour. Lateral and fore-and-aft oscillations damped out in a small number of cycles. It should be noted that the corresponding cable lengths on a full-scale towed airspeed head, twice the size of the one tested, would be twice as great. The corresponding speeds would be $\sqrt{2}$ times as great in order to maintain the same value of the Froude number $F = V^2/gl$.

When the model was drawn up to about 3 feet from the airplane, it was sufficiently stable at 80 miles per hour. Unstable oscillations did not start while the body was left in this position for about a minute. This behavior does not necessarily indicate that the oscillations would have damped out once they had started. The theory shows that a large number of oscillations is required to double amplitude; the body might, therefore, have to be towed for a considerable length of time before oscillations would become noticeable. Unfortunately, no means were available to start an oscillation.

As the speed was increased, the motion became less stable, until at 95 miles per hour increasing oscillations occurred. As predicted by the theory, both the fore-and-aft and lateral oscillations had periods close to the period of a simple pendulum. The lateral and fore-and-aft oscillations inevitably combined to cause the instrument to travel in an elliptical orbit. The direction of rotation was such that the instrument swung back as it came closest to the fuselage. Probably the increased velocity near the fuselage fed energy into the motion with each oscillation and caused a greater rate of increase in the amplitude than would have been predicted by the theory.

Several modifications of the model were tried in an effort to improve the stability. Two modifications appeared to improve the stability of the pendulum oscillation at short cable lengths. One of these changes consisted in shifting the pivot point rearward 1/2 inch, and the other consisted in equipping the model with a hinged rudder with weight behind the hinge line and viscous damping. These changes prevented the oscillation from appearing spontaneously as the speed was gradually increased from 80 to 140 miles per hour. A theoretical study indicates that these changes should have only secondary effects on stability. These tests are not considered to be a conclusive demonstration of the stability of the body because it is not known whether oscillations would have damped out once they were started.

Various other modifications that were tried resulted in unstable short-period oscillations of the body. These tests were made at a speed of 80 miles per hour. A forward shift of the pivot point caused a pitching oscillation. This motion was believed to be the result of elasticity of the cable and mass unbalance of the body and was similar in nature to flutter. A freely hinged rudder with weight behind the hinge line caused a short-period yawing oscillation. The use of an asymetrical vertical fin, extending only below the body, caused a short-period rocking motion of the body.

Another type of instability has been encountered on a few occasions when the full-size NACA airspeed head was lowered at the full length of the cable (approximately 200 ft). In one case in which this motion was observed, the airspeed head was lowered from the door of a twin-engine low-wing cabin monoplane. The head was steady at speeds below 150 miles per hour, but at this speed oscillations of about 3-foot wave length in the cable originated at the airplane and traveled down to the body. As the speed was increased to 165 miles per hour, the oscillations became very violent and caused a pitching motion of the body. The whipping action at the lower end of the cable eventually caused it to break. A metal sphere was later towed from the same airplane and the oscillations occurred as before. The oscillation was therefore not related to the aerodynamic characteristics of the body. It was believed to be caused by the action of unsteady air flow from the wing-fuselage juncture on the tow cable. The same airspeed head has been used without difficulty at much higher speeds on other airplanes.

Several relatively light, large-size towed bodies have been tested in flight. The pendulum oscillation of these bodies has never been known to become unstable, even when the body was raised or lowered from the airplane quite slowly. This behavior is in agreement with the theoretical prediction. These bodies have a small value of μ compared with that of the towed airspeed head; the unstable region at normal flying speeds is therefore very small.

DISCUSSION OF RESULTS

The theoretical and experimental investigations have shown that the pendulum motion of a towed body may become unstable when the body is drawn up close to the airplane. The theory shows that the instability is not serious because the amplitude of the oscillations increases very slowly. The maximum cable length at which unstable oscillations can occur may be reduced by reduction of the ratio of radius of gyration to tail length of the body and by increase of the fin area and aspect ratio.

More violent instability of the pendulum oscillation than would be predicted by the theory, as well as other types of instability, may be introduced by unsteadiness or lack of uniformity of the air flow in the region where the body is lowered from the airplane. Inasmuch as no practical method has been found to provide large damping of the pendulum oscillations when the body is close to the airplane, it is desirable to lower the body from a point where it is not subjected to these disturbing influences. A suitable location would probably be on the plane of symmetry of a twin-engine airplane, or on the wing of a single-engine airplane at a point outside the slipstream. It also appears desirable to lower and raise the body at low flying speed, because the unstable oscillations then increase in amplitude very slowly. If these precautions are taken, it should be possible to lower a towed body without attention on the part of the pilot. The only possibility for unstable oscillations to develop would be if the body were left for long periods suspended only a few feet below the airplane. Oscillations of the system at large cable lengths are rapidly damped because of the cable drag.

CONCLUSIONS

1. A theoretical study of the motion of a suspended body stabilized by fins showed that it had two modes of lateral oscillation with the following characteristics:

(a) Weather-vane oscillation

The weather-vane mode of oscillation was rapidly damped and had a period about equal to that of the instrument oscillating as a weather vane about a vertical axis through its center of gravity.

(b) Pendulum oscillation

The period of the pendulum mode of oscillation was about the same as that of a simple pendulum of length equal to the vertical distance of the body below the airplane. The oscillation was damped by the cable drag at large cable lengths but was unstable at short cable lengths. The rate of increase of amplitude in the unstable region was very small. It was found that the unstable region could be restricted to short cable lengths at normal airplane speeds by keeping the radius of gyration of the body small and increasing the fin area, aspect ratio, and tail length.

2. In flight tests, more violent instability of the pendulum motion was encountered than would have been expected from the theory and other types of

instability occasionally occurred. These conditions were attributed to the action of unsteady air flow on the cable. It is believed that unsatisfactory behavior of a towed suspended body can be avoided by lowering and raising the body at low flying speeds from a point on the airplane where the air flow is uniform.

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APPENDIX

DETERMINATION OF EQUIVALENT DRAG COEFFICIENT OF THE CABLE

The drag of each cable element of height dz is

$$dD_c = c_{d_c} w_c \frac{\rho}{2} V^2 dz$$

where c_{d_c} is the drag coefficient of the cable per unit vertical height. The variation of this drag coefficient with inclination of the cable has been obtained from the data of reference 3 and is presented in figure 8. If the assumption is made that the cable remains straight when viewed from the front, each cable element has a lateral velocity proportional to its distance below the airplane. The side force acting on each cable element is

$$dY_c = dD_c \frac{\dot{y}_c}{V}$$

$$= c_{d_c} w_c \frac{\rho}{2} V^2 \frac{\dot{Y}}{V} \frac{z}{Z} dz$$

The total side force is

$$Y_c = w_c \frac{\rho}{2} V \dot{y} \int_0^Z c_{d_c} \frac{z}{Z} dz$$

$$= w_c \frac{\rho}{2} V \dot{y} Z \int_0^1 c_{d_c} \frac{z}{Z} d\left(\frac{z}{Z}\right)$$

This side force has been determined by graphical integration for cables with various values of X/Z, the ratio of horizontal length to height. The cable form was assumed to be that of one-quarter of a sine wave, as shown in figure 9(a). Although the shape of the actual cable may deviate somewhat from a sine curve, the error in the calculated side force will be small.

The location of the resultant side force may also be determined graphically as the center of gravity of the area representing the side-force distribution. If the inertia of the cable is neglected, the lateral force will be balanced by reactions on the body and on the point of support; the magnitude of the reactions will depend on the position of the resultant side force on the cable. As shown by figure 9, most of the side force on the cable is transmitted to the body. Let the fraction of the total side force that is applied to the body be K. The side force applied to the body is then

$$Y_b = KY_c$$

$$= Kw_c \frac{\rho}{2}V\dot{y}Z\int_0^1 c_{d_c} \frac{z}{Z}\, d\left(\frac{z}{Z}\right)$$

$$\frac{dY_b}{d\dot{y}} = \left[\frac{Kw_cZ\int_0^1 c_{d_c} \frac{z}{Z}\, d\left(\frac{z}{Z}\right)}{S}\right]\frac{\rho}{2}VS$$

By comparison with formula (2), the quantity in brackets may be substituted as an equivalent drag coefficient in the formula for $Y_{\dot{y}}$. The relations may be summarized as follows:

$$Y_{\dot{y}} = \left(C_{D_b} + C_{D_c}\right)\frac{\frac{\rho}{2}VS}{m}$$

where

$$C_{D_c} = \frac{Kw_cZ}{S}\int_0^1 c_{d_c} \frac{z}{Z}\, d\left(\frac{z}{Z}\right)$$

The value of C_{D_c} may be determined from figure 4 for cables of various values of the ratio X/Z.

In order to determine the variation of C_{D_c} with cable length as the body is drawn up to the airplane, it is necessary to know how the ratio X/Z changes with cable length. Typical examples have been worked out for two airspeeds for the airspeed head and cable previously described. The shape of the cable was determined from consideration of the forces acting on the cable elements, obtained from reference 3. The cable shape for each speed is shown in figure 10. Any point on the cable may be considered as a point of suspension. The variation of the ratio X/Z as the cable is drawn in or let out may therefore be determined graphically from this figure.

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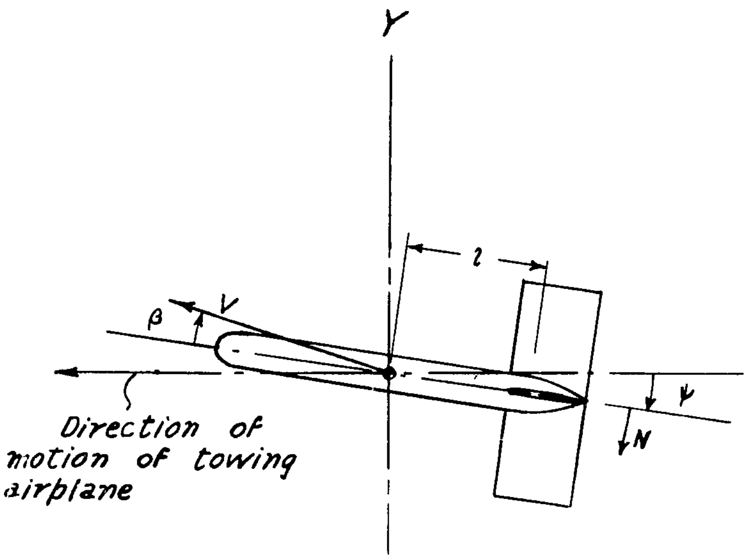

Figure 1.- Top view of trailing airspeed head showing symbols for the theory of lateral oscillations.

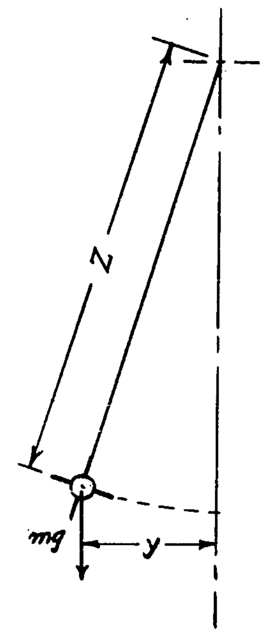

Figure 2.- Front view of trailing airspeed head.

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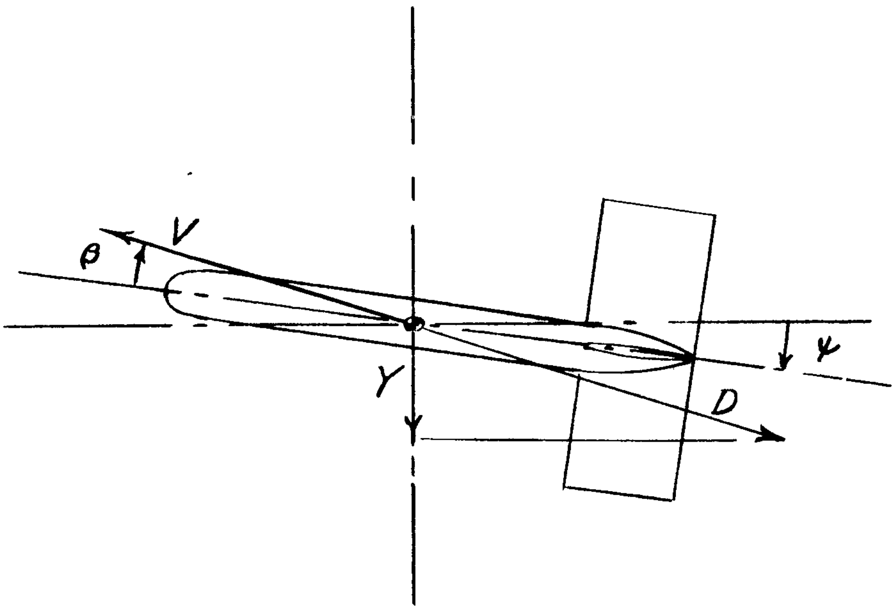

Figure 3.- Top view of trailing airspeed head showing component of side force due to drag.

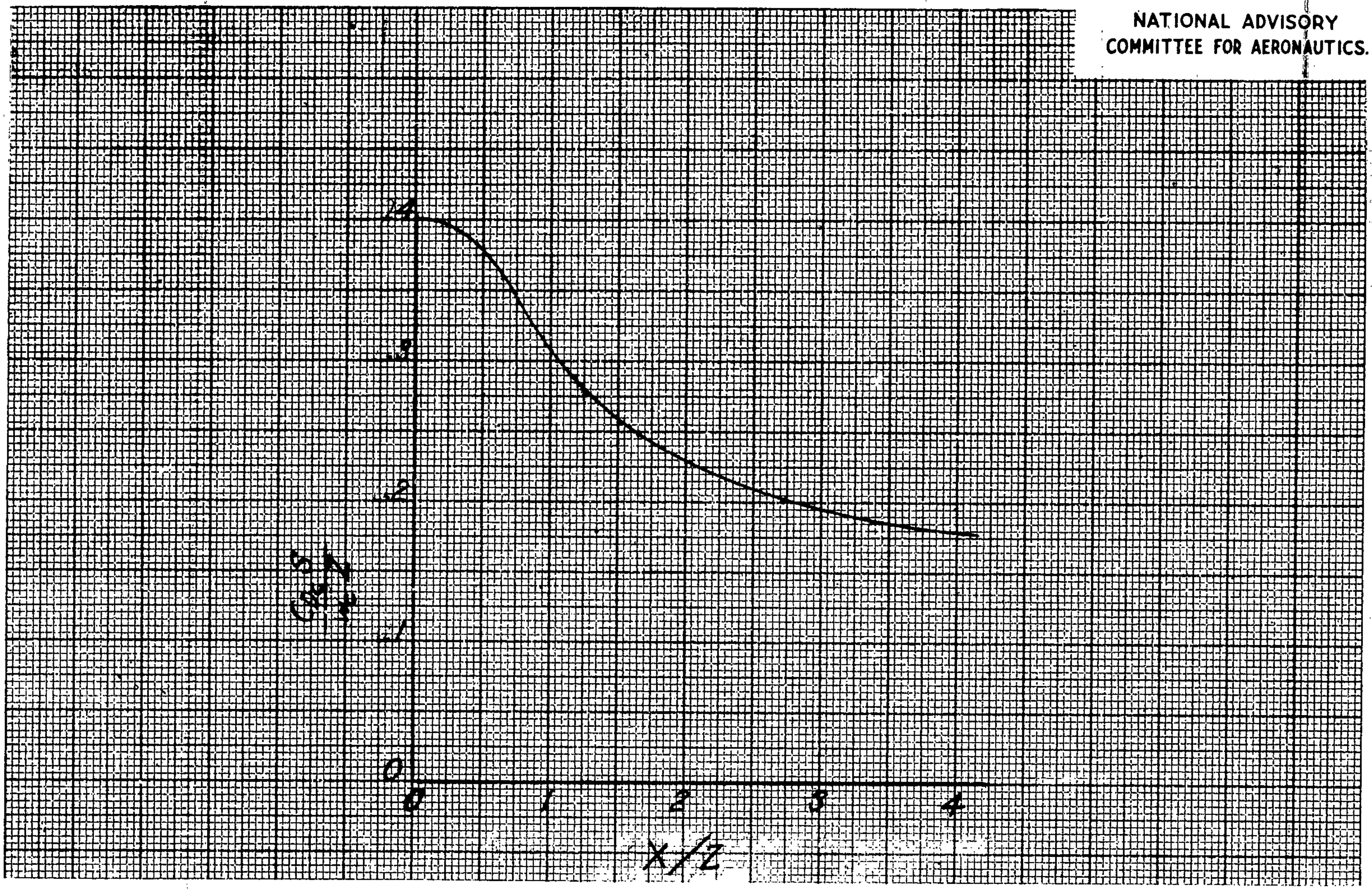

Figure 4.- Chart for determining equivalent drag coefficient of cable C_{D_c}.